MODEL PAPER, 2016

Bihar School Examination Board (Senior Secondary), Patna

Mathematics-XII

Set-I

Time : 3 Hours + 15 Minute] [*Maximum Marks* : 100

Instructions to the candidate :

1. Fill in your Roll No. in the space provided on the first page of this question paper.
2. This question paper consists of **40** objective type questions. Total marks allotted is **40**.
3. The candidate has to answer all the questions in the **OMR** Answer Sheet provided along with this question paper.
4. Before answering, the candidate has to ensure that the **OMR** Answer Sheet is available along with the question paper.
5. All entries must be confined to the area provided in the **OMR** Answer Sheet.
6. Answer all the questions by completely darkening the circles against the question numbers in the **OMR** Answer Sheet using Black/Blue Ball point pen only.
7. Do not fold or make any stray marks on the **OMR** Answer Sheet, failing which it would be difficult to evaluate the Answer Sheet.
8. Read all the instructions provided in the **OMR** Answer Sheet carefully before answering. After you finish answering, hand over the **OMR** Answer Sheet to the Invigilator. You are permitted to carry the **question paper only** along with you.

Section-I : Objective Type Questions

Time : 1 Hour 10 Minute] [*Full Marks* : 40

I. For the following Question Nos. **1** to **40** there is only one correct answer against each question. Mark the correct option on the answer sheet : $40 \times 1 = 40$

1. Derivative of x^x with respect to x is :
   (a) $x^x (\log x + 1)$ (b) x/x^{x-1} (c) $x.x^x$ (d) $(1 + \log x)$
2. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?
   (a) 2π cm/s (b) $0.7\ \pi$ cm/s (c) $1.4\ \pi$ cm/s (d) None of these
3. If $x = a \cos \theta, y = b \sin \theta$, then find $\frac{dy}{dx} = \ldots\ldots$
   (a) $\frac{-b}{a} \cot\theta$ (b) 0 (c) $\frac{b}{a} \tan\theta$ (d) $\frac{-b}{a} \tan\theta$
4. $\int \frac{1-\sin x}{\cos^2 x} dx$ is equal to :
   (a) $\tan x - \sec x + C$ (b) $\sec x - \tan x + C$ (c) $\tan x + \sec x + C$ (d) None of these
5. The value of $\int_0^{\pi/2} \log\left(\frac{4-3\sin x}{4+3\cos x}\right) dx$ is :
   (a) $\frac{3}{4}$ (b) 2 (c) 0 (d) $\frac{1}{4}$
6. If E and F are events such that P (E/F) = P (F/E) then :
   (a) P (E) = P (F) (b) E = F (c) $E \subset F$ but $E \neq F$ (d) $E \cap F = \phi$
7. If P and Q are symmetric matrices of same order, then PQ – QP is a :
   (a) Zero Matrix (b) Identity Matrix
   (c) Skew-symmetric Matrix (d) Symmetric Matrix
8. Let A be a square matrix of order 3 × 3, then $|KA|$ is equal to :
   (a) $3K|A|$ (b) $K^3|A|$ (c) $K|A|$ (d) $K^2|A|$

9. The parameter on which the value of the following determinant is not dependent :

$$\begin{vmatrix} 1 & m & m^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(b-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

(a) d (b) x (c) m (d) p

10. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is the co-factor of a_{ij} then D = ...

(a) $a_{11}A_{11} - a_{21}A_{21} + a_{31}A_{31}$ (b) $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$

(c) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$ (d) $A_{11} + A_{12} + A_{13}$

11. The area of the quadrilateral formed by the lines $y = 2x + 3$, $y = 0$, $x = 4$, $x = 6$ is :

(a) 26 square unit (b) 24 square unit

(c) 20 square unit (d) None of these

12. The degree of the diff. equation is :

$$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) + 7 = 0$$

(a) 2 (b) 1 (c) 3 (d) Not defined

13. $\int \frac{xe^x}{(1+x)^2}dx$ is equal to :

(a) $\frac{e^x}{1+x} + C$ (b) $\frac{-e^x}{(1+x)^2} + C$ (c) $e^x(x+1) + C$ (d) $\frac{e^x}{1+x^2} + C$

14. The direction ratio of a line are 2, 3, 7, then its direction cosines are :

(a) $\frac{1}{6}, \frac{1}{4}, \frac{7}{12}$ (b) $\sqrt{\frac{2}{62}}, \sqrt{\frac{3}{62}}, \sqrt{\frac{7}{62}}$ (c) $\frac{2}{\sqrt{62}}, \frac{3}{\sqrt{62}}, \frac{7}{\sqrt{62}}$ (d) $\frac{2}{12}, \frac{3}{12}, \frac{7}{12}$

15. $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y} =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{-3\pi}{4}$

16. Let A be a non-singular matrix of the order $n \times n$ then the $|\text{adj } A| =$ ......

(a) $n\,|A|$ (b) $|A|^{n-1}$ (c) $|A|$ (d) $|A|^n$

17. If A = {1, 2, 3}, B = {5, 6, 7} and $f : A \rightarrow B$ is a function such that $f(x) = x + 4$, then what type of a function is f ?

(a) many-one-onto (b) constant function

(c) one-one-onto (d) into

18. Let A = {1, 2, 3, 4 ..... n}. How many bijective function $f : A \rightarrow A$ can be defined ?

(a) $\frac{1}{2}(n!)$ (b) $(n-1)!$ (c) $n!$ (d) n

19. The point with position vectors (2, 6), (1, 2) and (p, 10) are collinear if the value of p is :

(a) 3 (b) – 3 (c) 12 (d) 6

20. The differential equation corresponding to curve $y = e^{p\cos^{-1}x}$ is :

(a) $(1-x^2)y^n - xy' - p^2y = 0$ (b) $(1-x^2)y'' - xy' + p^2y = 0$

(c) $\sqrt{1-x^2}\,y' = py$ (d) $(1-x^2)y'' + xy' - p^2y = 0$

21. A binary composition * is defined on R × R by $(a, b) * (c, d) = (ac, bc + d)$, where $a, b, c, d \in$ R. Find (2, 3) * (1, – 2).

(a) (1, 2) (b) (2, 1) (c) (1, 1) (d) (2, 2)

22. Find x and y if $(2x, x + y) = (6, 2)$.

(a) $x = 3, y = -1$ (b) $x = 1, y = 5$ (c) $x = -1, y = 3$ (d) $x = 5, y = 1$

23. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then the value of $xy + yz + zx$ is :

(a) – 1 (b) 1 (c) 0 (d) None of these

24. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) π

25. If $2\begin{bmatrix} x & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 6 & 6 \end{bmatrix}$ then the value of x and y are :

(a) $x = 2, y = 3$ (b) $x = 3, y = 2$ (c) $x = 2, y = 2$ (d) $x = 3, y = 3$

26. The values of a and b, when $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ are :

(a) $a = 1, b = -3$ (b) $a = -1, b = 3$ (c) $a = 1, b = 3$ (d) $a = -1, b = -3$

27. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then A^2 is :

(a) 27A (b) 2A (c) 3A (d) 1

28. The value of the determinant $\begin{vmatrix} 3 & 1 & 7 \\ 5 & 0 & 2 \\ 2 & 5 & 3 \end{vmatrix}$ is :

(a) 124 (b) 125 (c) 134 (d) 144

29. If $\begin{bmatrix} x+1 & x-1 \\ x-3 & x+2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$ then the value of x is :

(a) 1 (b) 2 (c) 3 (d) 4

30. If $\sqrt{x}+\sqrt{y}=5$ then $(4, a)$ at $\frac{dy}{dx} =$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{2}{3}$

31. If $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ then $\frac{dy}{dx} =$

(a) $\frac{1}{2(1+x^2)}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{2}{1+x^2}$ (d) None of these,

32. The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is :

(a) $(2\sqrt{2}, -1)$ (b) $(2\sqrt{2}, 0)$ (c) (0, 0) (d) (2, 2)

33. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ is equal to :

(a) $(x^{10} + 10^x)^{-1} + C$ (b) $10^x - x^{10} + C$ (c) $x^{10} + 10^x + C$ (d) $\log(x^{10} + 10^x) + C$

34. $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)dx =$

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 2

35. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is :

(a) $\frac{4}{3}(4\pi - \sqrt{3})$ (b) $\frac{4}{3}(4\pi + \sqrt{3})$ (c) $\frac{4}{3}(8\pi - \sqrt{3})$ (d) $\frac{4}{3}(8\pi + \sqrt{3})$

36. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$, when $y(1) = 1$ is :

(a) $y = \log x + x$ (b) $y = \log x + x^2$ (c) $y = xe^x - 1$ (d) $y = x\log x + x$

37. If a unit vector $\vec{a}$ may be angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then the value of θ is :

(a) 30° (b) 60° (c) 45° (d) 120°

[illegible] $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3. These lines will be coplanar if :

(a) $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$ (b) $\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = 0$

(c) $l_1 l_2 l_3 + m_1 m_2 m_3 + n_1 n_2 n_3 = 0$ (d) None of these

39. The corner points of the feasible region determined by the following system of linear inequalities :

$$2x + y \le 10$$
$$x + 3y \le 15$$
$$x, y \le 0$$

$2x + y \le 10, x + 3y \le 15, x, y \le 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $z = px + qy$, where $p, q > 0$, condition of p and q so that the maximum of z occurs at both (3, 4) and (0, 5) is :

(a) $p = q$ (b) $p = 2q$ (c) $p = 3q$ (d) $q = 3p$

40. If P(A/B) > P(A), then which of the following is correct :

(a) (P(B/A) < P(B) (b) $P(A \cap B) < P(A).P(B)$ (c) P(B/A) > P(B) (d) P(B/A) = P(B)

Answers

1. (a)	2. (c)	3. (a)	4. (a)	5. (c)	6. (a)	7. (c)	8. (b)	9. (d)	10. (c)	11. (a)	12. (d)	13. (a)	14. (c)
15. (b)	16. (b)	17. (c)	18. (c)	19. (a)	20. (d)	21. (b)	22. (a)	23. (b)	24.(c)	25. (a)	26. (a)	27. (c)	28. (c)
29. (b)	30. (c)	31. (a)	32. (a)	33. (d)	34. (b)	35. (c)	36. (d)	37. (b)	38. (a)	39. (d)	40. (c).		

Section-II : Non-Objective Type Questions

Time : 2 Hours 05 Minute] [*Full Marks* : 60

Instructions to the candidate :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. Section II of this question paper consists of **12** non-objective type questions having total marks **60**.
4. The candidate has to answer all the short answer type questions from Q. No. **1** to Q. No. **8** and all **4** Long answer type questions from Q. No. **9** to Q. No. **12** in his/her answer-book which is provided separately. Q. Nos. **1** to **8** carry **4** marks each and Q. Nos. **9** to **12** carry **7** marks each.
5. Write the Question number with every answer.

I. Question Nos. **1** to **8** are of Short answer type. Each question carries **4** marks : $8 \times 4 = 32$

1. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

[**Ans.** See Page No. 3, Q. No. 2.]

2. Prove that : $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

[**Ans.** See Page No. 12, Q. No. 1.]

3. If $2\begin{bmatrix} x & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 6 & 6 \end{bmatrix}$ then find the values of x and y.

[**Ans.** See Page No. 24, Q. No. 3.]

Or

Find the values of a and b for which : $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

[**Ans.** See Page No. 25, Q. No. 4.]

4. Using property of determinants, show that : $\begin{vmatrix} x+y & 2x & 2x \\ 2x & x+y & 2x \\ 2x & 2x & x+y \end{vmatrix} = (5x+y)(y-x)^2$

[**Ans.** See Page No. 37, Q. No. 2.]

5. If $y^x = e^{y-x}$, prove that : $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

[**Ans.** See Page No. 50, Q. No. 4.]

6. Find the intervals in which the function given by $f(x) = x^2 - 4x + 6$ is :

(a) strictly increasing

(b) strictly decreasing.

[**Ans.** See Page No. 65, Q. No. 4.]

7. Evaluate : $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx.$

[**Ans.** See Page No. 81, Q. No. 3.]

Or

Integrate the function $\frac{\cos x}{(1 - \sin x)(2 - \sin x)}$ with respect to x.

[**Ans.** See Page No. 82, Q. No. 4.]

8. Evaluate : $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

[**Ans.** See Page No. 91, Q. No. 1.]

Or

Evaluate : $\int_1^2 \frac{x e^x}{(1+x)^2} dx$

[**Ans.** See Page No. 92, Q. No. 3.]

II. Question No. **9** to **12** are of long answer type. Each question carries 7 marks. $4 \times 7 = 28$

9. A relation r on the set of straight lines is given by arb iff 'a is parallel to b'. Show that 'r' is an equivalence relation.

[**Ans.** See Page No. 5, Q. No. 2.]

10. Prove that : $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$

[**Ans.** See Page No. 17, Q. No. 5.]

Or

Using elementary operations, find inverse of the following matrix : $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

[**Ans.** See Page No. 29, Q. No. 9.]

11. Using properties of determinants, prove the following : $\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2$

[**Ans.** See Page No. 41, Q. No. 4.]

Or

Show that the function $f(x) = |x - 3|, x \in IR$ is continuous but not differentiable at $x = 3$.

[**Ans.** See Page No. 59, Q. No. 7.]

12. A wire of length 28 cm is to be cut into two pieces. One of the piece is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum ?

[**Ans.** See Page No. 71, Q. No. 2.]

Or

Evaluate : $\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx.$

[**Ans.** See Page No. 85, Q. No. 4.]

Set-II

Section-I : Objective Type Questions

I. General Instructions : See Set-I.

1. Consider the binary operation * on Q defined by $x * y = 1 + 12x + xy$, $\forall\, x, y \in Q$, then $2 * 3$ equals :

(a) 31 (b) 41 (c) 43 (d) 51

2. $f: A \to B$ will be an into function if :

(a) $f(A) \subset B$ (b) $f(A) = B$ (c) $B \subset f(A)$ (d) $f(B) \subset A$.

3. The range of the function $f(x) = \sqrt{(x-1)(3-x)}$ is :

(a) (1, 3) (b) (0, 1) (c) (– 2, 2) (d) None of these

4. If R be a relation on A such that A = {1, 2, 3}, and R = {(2, 2), (3, 3), (2, 3), (3, 2), (3, 1), (2, 1)}, then R is :

(a) reflexive (b) symmetric (c) equivalence (d) transitive

5. If $f: R \to R$ be defined as $f(x) = 2x + 3$, then $f^{-1}(x) =$:

(a) $2x - 3$ (b) $\frac{x-3}{2}$ (c) $\frac{x+3}{2}$ (d) None of these

6. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 9y = 0$ is :

(a) 2 (b) 3 (c) 4 (d) None of these

7. The differential equation of family of lines passing through the origin is :

(a) $x\frac{dy}{dx} = y$ (b) $y\frac{dy}{dx} = x$ (c) $\frac{dy}{dx} = y$ (d) $\frac{dy}{dx} = x$

8. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then :

(a) $|A| = 0$ (b) A^{-1} exists (c) $A^2 = 2A$ (d) None of these

9. If $y = \sec^{-1}\left[\frac{\sqrt{x}+1}{\sqrt{x}-1}\right] + \sin^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$ then $\frac{dy}{dx} =$

(a) 1 (b) π (c) $\frac{\pi}{2}$ (d) 0

10. If $y = x^2 + 3x + 4$, then the slope (gradient) of the normal to the curve at point (1, 1) is :

(a) 5 (b) $-\frac{1}{5}$ (c) 8 (d) $-\frac{1}{8}$

11. If $y = \sec(\tan^{-1} x)$ then $\frac{dy}{dx} =$

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $-\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) None of these

12. If $2x + 5y - 6z + 3 = 0$ be the quation of a plane, then the equation of any plane parallel to the given plane is :

(a) $3x + 5y - 6z + 3 = 0$ (b) $2x - 5y + 6z + 3 = 0$

(c) $2x + 5y - 6z + k = 0$ (d) None of these

13. Let A and B be two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{A}{B}\right) =$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

14. The solution of $\frac{dy}{dx} = 1 + x + y + xy$:

(a) $x - y = k(1 + xy)$ (b) $\log(1 + y) = x + \frac{x^2}{2} + k$ (c) $\log(1 + x) = y + \frac{y^2}{2} + k$ (d) None of these

15. If $\vec{a} = 2\vec{i} + 5\vec{j} + \vec{k}$ and $\vec{b} = 4\vec{i} - 2\vec{j} + \vec{k}$ then $\vec{a}.\vec{b} =$

(a) 0 (b) – 1 (c) 1 (d) 2

16. $\int_{-\pi/2}^{+\pi/2} \sin^9 x\, dx$ equals :

(a) -1 (b) 1 (c) 0 (d) None of these

17. The probability of getting a doublet with 2 dice is :

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{5}{36}$

18. If $2\vec{i}+\vec{j}+\vec{k}$, $6\vec{i}-\vec{j}+2\vec{k}$ and $14\vec{i}-5\vec{j}+4\vec{k}$ be the position vectors of the points A, B and C respectively, then :

(a) A, B, C are collinear (b) A, B, C are non-collinear

(c) $\vec{AB} \perp \vec{BC}$ (d) None of these

19. The value of c in Rolle's theorem for $f(x) = x^2 - 1$ in interval $[-1, 1]$ is :

(a) $\frac{1}{2}$ (b) 0 (c) $\frac{1}{4}$ (d) None of these

20. The angle which the tangent to the curve $y = x^2$ at $(0, 0)$ makes with the positive direction of x-axis is :

(a) 90° (b) 0° (c) 45° (d) 30°

21. The relation 'less than' in the set of natural numbers is :

(a) only reflexive (b) only symmetric

(c) equivalence relation (d) only transitive

22. If function $f : R \to R$ be defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then $fof(x)$ is :

(a) $x^{\frac{1}{3}}$ (b) x^3 (c) $(3 - x^3)$ (d) x

23. Algebraic expression for $\sin(\cot^{-1} x)$ is :

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{\sqrt{1+x^2}}$ (c) $\frac{x}{\sqrt{1+x^2}}$ (d) None of these

24. The value of $\sec^2(\tan^{-1} 4) + \operatorname{cosec}^2(\cot^{-1} 2)$ is :

(a) 12 (b) 18 (c) 22 (d) 25

25. Scalar matrix is :

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

26. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ then $(A - 2I)(A - 3I)$ is equal to :

(a) O (b) A (c) I (d) 2A

27. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then the values of x and y are :

(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) $-1, -1$

28. If w is cube root of unity then the value of $\begin{vmatrix} 1 & w^6 & w^8 \\ w^6 & w^3 & w^7 \\ w^8 & w^7 & 1 \end{vmatrix}$ is :

(a) 1 (b) 2 (c) 3 (d) 4

29. If on [0, 1] Lagrange's Theorem is true for $f(x) = x^3 - 2x^2 - x + 3$, then the value of c is :

(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$

30. If $f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2 & \text{if } x \ge 3 \end{cases}$ then the value of $f(-3)$ is :

(a) 0 (b) -3 (c) 3 (d) 6

31. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

(a) (1, 2) (b) (2, 1) (c) (0, 0) (d) (2, 2)

32. The value of $\int \frac{dx}{x^2 + 2x + 2}$ is :

(a) $\tan^{-1}(x + 1) + C$ (b) $\tan^{-1}(x + 2) + C$ (c) $\sin^{-1}(x + 1) + C$ (d) $\sin^{-1}(x + 2) + C$

33. $\int_{-\pi\ 2}^{\pi\ 2} \sin^2 x \ dx =$

(a) $\frac{\pi}{4}$ (b) 2π (c) π (d) $\frac{\pi}{2}$

34. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

35. The differential equation for different values of A and B is the curve $y = Ae^x + Be^{-x}$ is :

(a) $\frac{d^2y}{dx^2} - 2y$ (b) $\frac{d^2y}{dx^2} = y$ (c) $\frac{d^2y}{dx^2} = 4y + 3$ (d) $\frac{d^2y}{dx^2} + y = 0$

36. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then the value of $x + y + z$ is :

(a) 0 (b) 3 (c) – 3 (d) 1

37. The coordinates of the point, which is equidistant from the points (0, 0, 0), $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ are :

(a) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{c}\right)$ (b) $\left(-\frac{a}{2}, -\frac{b}{2}, \frac{c}{2}\right)$ (c) $\left(\frac{a}{2}, -\frac{b}{2}, -\frac{c}{2}\right)$ (d) $\left(-\frac{a}{2}, \frac{b}{2}, -\frac{c}{2}\right)$

38. The maximum value of $Z = 4x + 2y$ subjected to the constraints $2x + 3y \le 18$, $x + y \ge 10$; $x, y \ge 0$ is :

(a) 39 (b) 54 (c) 15 (d) None of these

39. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is :

(a) $5C_4\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (c) $5C_1 \frac{1}{5}\left(\frac{4}{5}\right)^4$ (d) None of these

40. $\int_{-a}^{a} x\sqrt{a^2 - x^2} \ dx =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{8}$ (d) 0

Answers

1. (a)	2. (a)	3. (b)	4. (b)	5. (b)	6. (a)	7. (a)	8. (b)	9. (d)	10. (b)	11. (d)	12. (c)	13. (d)	14. (b)
15. (b)	16. (c)	17. (b)	18. (a)	19. (b)	20. (b)	21. (d)	22. (d)	23. (b)	24. (c)	25. (d)	26. (a)	27. (a)	28. (c)
29. (d)	30. (d)	31. (a)	32. (a)	33. (d)	34. (b)	35. (b)	36. (a)	37. (a)	38. (d)	39. (a)	40. (d).		

Section-II : Non-Objective Type Questions

I. General Instructions : See Set-I.

1. 5. What is the range of the function $f(x) = \frac{|x - 1|}{x - 1}$?

[**Ans.** See Page No. 3, Q. No. 5.]

2. Write the function of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $x < \pi$ in the simplest form.

[**Ans.** See Page No. 13, Q. No. 4.]

Or

Solve the following matrix equation for x : $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$

[**Ans.** See Page No. 25, Q. No. 6.]

3. Show that $\Delta = \Delta_1$ where : $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$

[**Ans.** See Page No. 38, Q. No. 5.]

Or

Test the continuity of the function $f(x)$ at $x = 0$, where

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$$

[**Ans.** See Page No. 51, Q. No. 5.]

4. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is :
(a) Strictly increasing
(b) Strictly decreasing.
[**Ans.** See Page No. 66, Q. No. 6.]

Or

Integrate the function $\dfrac{\cos x}{(1 - \sin x)(2 - \sin x)}$ with respect to x.

[**Ans.** See Page No. 82, Q. No. 4.]

5. Evaluate : $\int_0^{2\pi} \dfrac{1}{1+e^{\sin x}} dx$
[**Ans.** See Page No. 92, Q. No. 5.]

Or

Using integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.
[**Ans.** See Page No. 105, Q. No. 3.]

6. Find the particular solution of the differential equation :

$$\frac{dy}{dx} = 1 + x + y + xy, \text{ given that } y = 0 \text{ when } x = 1.$$

[**Ans.** See Page No. 115, Q. No. 5.]

7. Solve the following differential equation : $x\dfrac{dy}{dx} = y - x \tan\dfrac{y}{x}$

[**Ans.** See Page No. 117, Q. No. 11.]

8. Show that the line $\dfrac{x+3}{-3} = \dfrac{y-1}{1} = \dfrac{z-9}{5}$ and $\dfrac{x+1}{-1} = \dfrac{y-2}{2} = \dfrac{z-5}{5}$ are coplanar.

[**Ans.** See Page No. 147, Q. No. 4.]

II. *General Instructions* : See Set-I.

9. If the function $f: R \rightarrow R$ is given by $f(x) = x^2 + 2$ and $g: R \rightarrow R$ is given by $g(x) : \dfrac{x}{x-1}$; $x \neq 1$, then find fog and gof and hence find $fog\,(2)$ and $gof\,(-3)$.
[**Ans.** See Page No. 6, Q. No. 7.]

Or

Prove that : $\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{5} + \tan^{-1}\dfrac{1}{7} + \tan^{-1}\dfrac{1}{8} = \dfrac{\pi}{4}$

[**Ans.** See Page No. 18, Q. No. 9.]

10. Solve by matrix method the following system of equations :

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

[**Ans.** See Page No. 42, Q. No. 7.]

11. If the following function $f(x)$ is continuous at $x = 0$, find the value of a.

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & ; \quad x < 0 \\ a & ; \quad x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+x}} & ; \quad x > 0 \end{cases}$$

[**Ans.** See Page No. 58, Q. No. 4.]

Or

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\dfrac{4r}{3}$. Also show that the maximum volume of the cone is $\dfrac{8}{27}$ of the volume of the sphere.

[**Ans.** See Page No. 73, Q. No. 6.]

12. Evaluate : $\int_0^1 (3x^2 + 1)\,dx$ as a limit of a sum.

[**Ans.** See Page No. 95, Q. No. 1.]

❖❖

Instructions :

1. All entries should be confined to the area provided.
2. In the OMR Answer Sheet the Question Nos. progress from top to bottom.
3. For marking answers, use BLACK/BLUE BALL POINT PEN ONLY.
4. Mark your Roll No., Roll Code No., Name of Exam. Centre in the boxes/space provided in the OMR Answer Sheet.
5. Fill in your Name, Signature, Subject, Date of Exam in the space provided in the OMR Answer Sheet.
6. Mark your Answer by darkening the CIRCLE completely, like this.

Correct Method

Ⓐ ● Ⓒ Ⓓ

Wrong Method

Ⓐ ◓ Ⓒ Ⓓ
Ⓐ ◐ Ⓒ Ⓓ
Ⓐ ⊖ Ⓒ Ⓓ

7. Do not fold or make any stray marks in the OMR Answer Sheet.
8. If you do not follow the instructions given above, it may be difficult to evaluate the Answer Sheet. Any resultant loss on the above account i.e., not following the instructions completely shall be of the candidates only.

निर्देश :

1. सभी प्रविष्टियाँ दिये गये स्थान तक ही सीमित रखें।
2. OMR उत्तर पत्र में प्रश्न संख्या क्रमशः ऊपर से नीचे की ओर दी गई है।
3. उत्तर केवल काले/नीले बॉल प्वाइंट पेन द्वारा चिह्नित करें।
4. अपना रोल नं०, रोल कोड नं०, परीक्षा केन्द्र का नाम OMR उत्तर पत्र पर निर्दिष्ट खाली/स्थानों में/पर लिखें।
5. OMR उत्तर पत्र में निर्धारित स्थान पर अपना नाम, हस्ताक्षर, विषय, परीक्षा का दिनांक की पूर्ति करें।
6. अपने उत्तर के घेरे को पूर्ण रूप से प्रगाढ़ करते हुए चिह्नित करें।

सही विधि

Ⓐ ● Ⓒ Ⓓ

गलत विधियाँ

Ⓐ ◓ Ⓒ Ⓓ
Ⓐ ◐ Ⓒ Ⓓ
Ⓐ ⊖ Ⓒ Ⓓ

7. OMR उत्तर पत्र को न मोड़ें अथवा उस पर जहाँ-तहाँ चिह्न न लगाएँ।
8. ऊपर दिये गये निर्देशों का पालन न किए जाने की स्थिति में उत्तर पत्रों का मूल्यांकन करना कठिन होगा। ऐसे में नतीजे की दृष्टि से किसी भी प्रकार की क्षति का जिम्मेदार केवल परीक्षार्थी होगा।

1. Name (in BLOCK letters)/नाम (छापे के अक्षर में)

2. Date of Exam/परीक्षा की तिथि

3. Subject/विषय

4. Name of the Exam Centre/परीक्षा केन्द्र का नाम

5. Full Signature of Candidate/परीक्षार्थी का पूर्ण हस्ताक्षर

6. Invigilator's Signature/निरीक्षक का हस्ताक्षर

7. Roll Code / रोल कोड

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

8. Roll Number / रोल नं०

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

For answering darken the circles given below/उत्तर के लिए नीचे अंकित घेरे को प्रगाढ़ करें।

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D
11.	A	B	C	D
12.	A	B	C	D
13.	A	B	C	D
14.	A	B	C	D
15.	A	B	C	D
16.	A	B	C	D
17.	A	B	C	D
18.	A	B	C	D
19.	A	B	C	D
20.	A	B	C	D
21.	A	B	C	D
22.	A	B	C	D
23.	A	B	C	D
24.	A	B	C	D
25.	A	B	C	D
26.	A	B	C	D
27.	A	B	C	D
28.	A	B	C	D
29.	A	B	C	D
30.	A	B	C	D
31.	A	B	C	D
32.	A	B	C	D
33.	A	B	C	D
34.	A	B	C	D
35.	A	B	C	D
36.	A	B	C	D
37.	A	B	C	D
38.	A	B	C	D
39.	A	B	C	D
40.	A	B	C	D

EXAMINATION PAPER, 2015

Bihar School Examination Board (Senior Secondary), Patna

Mathematics

Class-XII

Time : 3 Hours 15 Minutes] [*Full Marks* : 100

Instructions to the candidate :

1. Fill in your Roll No. in the space provided on the first page of this question paper.
2. This question paper consists of **40** objective type questions. Total marks allotted is **40**.
3. The candidate has to answer all the questions in the **OMR** Answer Sheet provided along with this question paper.
4. Before answering, the candidate has to ensure that the **OMR** Answer Sheet is available along with the question paper.
5. All entries must be confined to the area provided in the **OMR** Answer Sheet.
6. Answer all the questions by completely darkening the circles against the question numbers in the **OMR** Answer Sheet using Black/Blue Ball point pen only.
7. Do not fold or make any stray marks on the **OMR** Answer Sheet, failing which it would be difficult to evaluate the Answer Sheet.
8. Read all the instructions provided in the **OMR** Answer Sheet carefully before answering. After you finish answering, hand over the **OMR** Answer Sheet to the Invigilator. You are permitted to carry the **question paper only** along with you.

Section-I

Objective Type Questions

Time : 1 Hour 10 Minutes] [*Full Marks* : 40

I. For the following Question Nos. **1** to **40** there is only one correct answer against each question. Mark the correct option on the answer sheet : 40 × 1 = 40

1. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in A$, then the function $f : A \to B$ is :

(a) One-One (b) Constant (c) Onto (d) Many One

2. The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

3. $\tan^{-1}x =$

(a) $\cot^{-1}x$ (b) $\frac{1}{\cot^{-1}x}$ (c) $\cot^{-1}\frac{1}{x}$ (d) $-\cot^{-1}x$

4. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

(a) $(x-y)(y+z)(z+x)$ (b) $(x+y)(y-z)(z-x)$ (c) $(x-y)(y-z)(z+x)$ (d) $(x-y)(y-z)(z-x)$

5. If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$, then $x =$:

(a) 3 (b) 4 (c) 5 (d) 8

6. $\frac{d}{dx}\left(\sin^{-1}x\right) =$

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$ (c) $2(1-x^2)$ (d) $(1-x^2)$

7. $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) =$

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{\sqrt{1-x^2}}$

8. If $y = \sin(x^3)$, then $\frac{dy}{dx}$ = :

(a) $x^3 \cos(x^3)$ (b) $3x^2 \sin(x^3)$ (c) $3x^2 \cos(x^3)$ (d) $\cos(x^3)$

9. If $y = \tan^2 x$, then $\frac{dy}{dx}$ = :

(a) $\sec^2 x$ (b) $\sec^4 x$ (c) $2\tan x \sec x$ (d) $2\tan x \sec^2 x$

10. $\int 1.dx =$

(a) $x + k$ (b) $1 + k$ (c) $\frac{x^2}{2} + k$ (d) $\log x + k$

11. $\int \frac{dx}{\sqrt{x}} =$

(a) $\sqrt{x} + k$ (b) $2\sqrt{x} + k$ (c) $x + k$ (d) $\frac{2}{3}x^{3/2} + k$

12. $\int \frac{dx}{1+\cos x} =$

(a) $\tan\frac{x}{2} + k$ (b) $\frac{1}{2}\tan\frac{x}{2} + k$ (c) $2\tan\frac{x}{2} + k$ (d) $\tan^2\frac{x}{2} + k$

13. $\int_a^b x^5\, dx =$

(a) $b^5 - a^5$ (b) $\frac{b^6 - a^6}{6}$ (c) $\frac{a^6 - b^6}{6}$ (d) $a^5 - b^5$

14. The Solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is :

(a) $x - y = k$ (b) $x^2 - y^2 = k$ (c) $x^3 - y^3 = k$ (d) $xy = k$

15. The integrating factor of the linear differential equation $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ is :

(a) $\tan x$ (b) $e^{\tan x}$ (c) $\log \tan x$ (d) $\tan^2 x$

16. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$ is :

(a) 1 (b) 2 (c) 3 (d) 4

17. The order of the differential equation $\frac{d^2y}{dx^2} + x^3\left(\frac{dy}{dx}\right)^3 = x^4$ is :

(a) 1 (b) 2 (c) 3 (d) 4

18. The position vector of the point (1, 0, 2) is :

(a) $\vec{i} + \vec{j} + 2\vec{k}$ (b) $\vec{i} + 2\vec{j}$ (c) $\vec{i} + 3\vec{k}$ (d) $\vec{i} + 2\vec{k}$

19. The modulus of $7\vec{i} - 2\vec{j} + \vec{k}$ is :

(a) $\sqrt{10}$ (b) $\sqrt{55}$ (c) $3\sqrt{6}$ (d) 6

20. If 0 be the origin and $\vec{OP} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{OQ} = 5\hat{i} + 4\hat{j} - 3\hat{k}$, then $\vec{PQ}$ is equal to :

(a) $7\hat{i} + \hat{j} - 7\hat{k}$ (b) $-3\hat{i} - \hat{j} - \hat{k}$ (c) $-7\hat{i} - 7\hat{j} + 7\hat{k}$ (d) $3\hat{i} + \hat{j} + \hat{k}$

21. The scalar product of $5\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} - 4\hat{j} + 7\hat{k}$ is :

(a) 10 (b) – 10 (c) 15 (d) – 15

22. If $\vec{a} \cdot \vec{b} = 0$, then :

(a) $\vec{a} \perp \vec{b}$ (b) $\vec{a} \parallel \vec{b}$ (c) $\vec{a} + \vec{b} = \vec{0}$ (d) $\vec{a} - \vec{b} = 0$

23. $\vec{i} \cdot \vec{j} =$

(a) 0 (b) 1 (c) $\vec{k}$ (d) $-\vec{k}$

24. $\vec{k} \times \vec{j} =$

(a) 0 (b) 1 (c) $\vec{i}$ (d) $-\vec{i}$

25. $\vec{a} \cdot \vec{a}$

(a) 0 (b) 1 (c) $|\vec{a}|^2$ (d) $|\vec{a}|$

26. The direction cosines of the y-axis are :

(a) (0, 0, 0) (b) (1, 0, 0) (c) (0, 1, 0) (d) (0, 0, 1)

27. The directions ratios of the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are :

(a) $x_1 + x_2, y_1 + y_2, z_1 + z_2$ (b) $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(c) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$ (d) $x_2 - x_1, y_2 - y_1, z_2 - z_1$

28. The coordinates of the midpoint of the line segment joining the points (2, 3, 4) and (8, – 3, 8) are :

(a) (10, 0, 12) (b) (5, 6, 0) (c) (6, 5, 0) (d) (5, 0, 6)

29. If the direction cosines of two straight lines are l_1, m_1, n_1 and l_2, m_2, n_2 then the cosine of the angle θ between them or $\cos\theta$ is :

(a) $(l_1 + m_1 + n_1)(l_2 + m_2 + n_2)$ (b) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(c) $l_1l_2 + m_1m_2 + n_1n_2$ (d) $\frac{l_1 + m_1 + n_1}{l_2 + m_2 + n_2}$

30. The direction ratios of the normal to the plane $7x + 4y - 2z + 5 = 0$ are :

(a) 7, 4, 5 (b) 7, 4, – 2 (c) 7, 4, 2 (d) 0, 0, 0

31. If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$, then :

(a) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$ (b) $al + bm + cn = 0$ (c) $al^2 + bm^2 + cn^2 = 0$ (d) $a^2l^2 + b^2m^2 + c^2n^2 = 0$

32. If the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2x + d_2 = 0$ are perpendicular to each other, then :

(a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} + \frac{b_1}{b_2} + \frac{c_1}{c_2} = 0$ (c) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (d) $a_1^2a_2^2 + b_1^2b_2^2 + c_1^2c_2^2 = 0$

33. The distance of the plane $2x - 3y + 6z + 7 = 0$ from the point (2, – 3, – 1) is :

(a) 4 (b) 3 (c) 2 (d) $\frac{1}{5}$

34. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B) =$:

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$

35. If A and B are two independent events, then :

(a) $P(A \cup B) = 1 - P(A')P(B')$ (b) $P(A \cap B) = 1 - P(A')P(B')$

(c) $P(A \cup B) = 1 + P(A')P(B')$ (d) $P(A \cup B) = \frac{P(A')}{P(B')}$

36. $\begin{vmatrix} 3 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{vmatrix} =$

(a) 40 (b) 50 (c) 42 (d) 15

37. The inverse of $A = \begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ will not be obtained if k has the value :

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{15}{2}$

38. $\cos^{-1}\dfrac{1-x^2}{1+x^2} =$

(a) $2\cos^{-1}x$ (b) $2\sin^{-1}x$ (c) $2\tan^{-1}x$ (d) $\cos^{-1}2x$

39. For any unit matrix l :

(a) $l^2 = I$ (b) $|I| = 0$ (c) $|I| = 2$ (d) $|I| = 5$

40. If, $x > a$ $\int\dfrac{dx}{x^2-a^2} =$:

(a) $\dfrac{1}{2a}\log\dfrac{x-a}{x+a}+k$ (b) $\dfrac{1}{2a}\log\dfrac{x+a}{x-a}+k$ (c) $\dfrac{1}{a}\log(x^2-a^2)+k$ (d) $\log(x+\sqrt{x^2-a^2})+k$

Section-II
Non-Objective Type Questions

Time : 2 Hours 05 Minutes] [*Full Marks* : 60

Instructions to the candidate :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. Section II of this question paper consists of **12** non-objective type questions having total marks **60**.
4. The candidate has to answer all the short answer type questions from Q. No. **1** to Q. No. **8** and all **4** Long answer type questions from Q. No. **9** to Q. No. **12** in his/her answer-book which is provided separately. Q. Nos. **1** to **8** carry **4** marks each and Q. Nos. **9** to **12** carry **7** marks each.
5. Write the Question number with every answer.

I. Question Nos. **1** to **8** are of Short answer type. Each question carries **4** marks : $8 \times 4 = 32$

1. Prove that : $4(\cot^{-1}3 + \text{coses}^{-1}\sqrt{5}) = \pi$

2. Evaluate $A = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

3. Find the values of X and Y : $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

4. If $y = \sin[\cos\{\tan(\cot x)\}]$, then find $\dfrac{dy}{dx}$.

5. If $y = \tan^{-1}x$, then find $\dfrac{dy}{dx}$ by first principle.

6. Integrate : $\int e^x \cos x\, dx$.

7. If $\vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} - 6\hat{j} + 8\hat{k}$, find $\vec{a} \times \vec{b}$.

8. A speaks the truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact ?

II. Question Nos. **9** to **12** are of Long answer type. Each question carries **7** marks. $4 \times 7 = 28$

9. Evaluate : $\int_0^{\pi/4} \sin^2 x\, dx$.

10. Solve the differential equation : $(x - y)\,dy - (x + y)\,dx = 0$.

Or

Solve the differential equation : $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{y^2}{x^2}$

11. Find the equations of the straight line perpendicular to the two lines $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$; $\dfrac{x}{1} = \dfrac{y-7}{-3} = \dfrac{z+7}{2}$ and passing through their point of intersection.

12. Minimize $Z = -3x + 3y$

Subject to :

$x + 2y \le 8$

$3x + 2y \le 12$

$x \ge 0, y \ge 0$.

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OMR No.

Instructions :

1. All entries should be confined to the area provided.
2. In the OMR Answer Sheet the Question Nos. progress from top to bottom.
3. For marking answers, use BLACK/BLUE BALL POINT PEN ONLY.
4. Mark your Roll No., Roll Code No., Name of Exam. Centre in the boxes/space provided in the OMR Answer Sheet.
5. Fill in your Name, Signature, Subject, Date of Exam in the space provided in the OMR Answer Sheet.
6. Mark your Answer by darkening the CIRCLE completely, like this.

Correct Method

Ⓐ ● Ⓒ Ⓓ

Wrong Method

Ⓐ ◓ Ⓒ Ⓓ
Ⓐ ◐ Ⓒ Ⓓ
Ⓐ ⊖ Ⓒ Ⓓ

7. Do not fold or make any stray marks in the OMR Answer Sheet.
8. If you do not follow the instructions given above, it may be difficult to evaluate the Answer Sheet. Any resultant loss on the above account i.e., not following the instructions completely shall be of the candidates only.

निर्देश :

1. सभी प्रविष्टियाँ दिये गये स्थान तक ही सीमित रखें।
2. OMR उत्तर पत्र में प्रश्न संख्या क्रमशः ऊपर से नीचे की ओर दी गई है।
3. उत्तर केवल काले/नीले बॉल प्वाइंट पेन द्वारा चिह्नित करें।
4. अपना रोल नं०, रोल कोड नं०, परीक्षा केन्द्र का नाम OMR उत्तर पत्र पर निर्दिष्ट खाली/स्थानों में/पर लिखें।
5. OMR उत्तर पत्र में निर्धारित स्थान पर अपना नाम, हस्ताक्षर, विषय, परीक्षा का दिनांक की पूर्ति करें।
6. अपने उत्तर के घेरे को पूर्ण रूप से प्रगाढ़ करते हुए चिह्नित करें।

सही विधि

Ⓐ ● Ⓒ Ⓓ

गलत विधियाँ

Ⓐ ◓ Ⓒ Ⓓ
Ⓐ ◐ Ⓒ Ⓓ
Ⓐ ⊖ Ⓒ Ⓓ

7. OMR उत्तर पत्र को न मोड़ें अथवा उस पर जहाँ–तहाँ चिह्न न लगाएँ।
8. ऊपर दिये गये निर्देशों का पालन न किए जाने की स्थिति में उत्तर पत्रों का मूल्यांकन करना कठिन होगा। ऐसे में नतीजे की दृष्टि से किसी भी प्रकार की क्षति का जिम्मेदार केवल परीक्षार्थी होगा।

1. Name (in BLOCK letters)/नाम (छापे के अक्षर में)
2. Date of Exam/परीक्षा की तिथि
3. Subject/विषय
4. Name of the Exam Centre/परीक्षा केन्द्र का नाम
5. Full Signature of Candidate/परीक्षार्थी का पूर्ण हस्ताक्षर
6. Invigilator's Signature/निरीक्षक का हस्ताक्षर

7. Roll Code / रोल कोड

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

8. Roll Number / रोल नं०

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

For answering darken the circles given below/उत्तर के लिए नीचे अंकित घेरे को प्रगाढ़ करें।

1.	A	B	C	D	14.	A	B	C	D	27.	A	B	C	D
2.	A	B	C	D	15.	A	B	C	D	28.	A	B	C	D
3.	A	B	C	D	16.	A	B	C	D	29.	A	B	C	D
4.	A	B	C	D	17.	A	B	C	D	30.	A	B	C	D
5.	A	B	C	D	18.	A	B	C	D	31.	A	B	C	D
6.	A	B	C	D	19.	A	B	C	D	32.	A	B	C	D
7.	A	B	C	D	20.	A	B	C	D	33.	A	B	C	D
8.	A	B	C	D	21.	A	B	C	D	34.	A	B	C	D
9.	A	B	C	D	22.	A	B	C	D	35.	A	B	C	D
10.	A	B	C	D	23.	A	B	C	D	36.	A	B	C	D
11.	A	B	C	D	24.	A	B	C	D	37.	A	B	C	D
12.	A	B	C	D	25.	A	B	C	D	38.	A	B	C	D
13.	A	B	C	D	26.	A	B	C	D	39.	A	B	C	D
										40.	A	B	C	D

MODEL PAPER, 2016

Central Board of Secondary Education (CBSE), All India
Mathematics-XII
(Solved)

Time : 3 Hours] [*Max. Marks* : 100

General Instructions :

(i) *All* questions are compulsory.
(ii) Please check that this question paper contains **26** questions.
(iii) Questions **1–6** in Section A are very short-answer type questions carrying **1** mark each.
(iv) Questions **7–19** in Section B are long-answer I type questions carrying **4** marks each.
(v) Questions **20–26** in Section C are long-answer II type questions carrying **6** marks each.
(vi) Please write down the serial number of the question before attempting it.

Section-A

Question numbers **1** to **6** carry **1** mark each.

1. If $f : R \to R$, be defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$.
   [**Ans.** : See Page No. 3, Q. No. 4.]
2. A binary composition $*$ is defined in Z^+ by $a * b = a^b - a$; $a, b \in Z^+$. Find $2 * 5$, where Z^+ is the set of positive integers.
   [**Ans.** : See Page No. 5, Q. No. 15.]
3. Write the principal value of $[\tan^{-1}(-\sqrt{3}) + \tan^{-1}(1)]$
   [**Ans.** : See Page No. 11, Q. No. 10.]
4. Find $y = \frac{1}{\sin x} + e^x$, then find $\frac{dy}{dx}$.
   [**Ans.** : See Page No. 49, Q. No. 3.]
5. $\int \frac{d}{dx} (\log_e^x)\, dx = \ldots\ldots + k$, where k is a constant.
   [**Ans.** : See Page No. 79, Q. No. 2.]
6. If $f(x) = \int_0^x t \sin t\, dt$, then write the value of $f'(x)$.
   [**Ans.** : See Page No. 90, Q. No. 1.]

Section-B

Question numbers **7** to **19** carry **4** marks each.

7. If $\begin{bmatrix} 2x & 4 \end{bmatrix} \begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, find the positive value of x.
   [**Ans.** : See Page No. 25, Q. No. 8.]
8. Express the determinant $|A|$ in factors, where :

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

[**Ans.** : See Page No. 37, Q. No. 3.]

Or

Test the continuity of the function $f(x)$ at $x = 0$, where

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

[**Ans.** : See Page No. 51, Q. No. 5.]

9. Using properties of determinants, show that :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

[**Ans.** : See Page No. 40, Q. No. 10.]

10. Find the intervals in which the function given by $f(x) = x^2 - 4x + 6$ is :
(a) strictly increasing (b) strictly decreasing.
[**Ans.** : See Page No. 65, Q. No. 4.]

11. Evaluate : $\displaystyle\int \frac{dx}{\sqrt{\sin^3 x \sin (x+a)}}$.

[**Ans.** : See Page No. 81, Q. No. 2.]

12. Evaluate : $\displaystyle\int_0^4 \{|x| + |x-2| + |x-4|\}\, dx$

[**Ans.** : See Page No. 95, Q. No. 14.]

13. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.
[**Ans.** : See Page No. 103, Q. No. 4.]

14. Find the particular solution of the differential equation : $\dfrac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$.

[**Ans.** : See Page No. 115, Q. No. 5.]

Or

Find the vector and cartesian equation of the line passing through the point (2, 1, 3) and perpendicular to the lines $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ and $\dfrac{x}{-3} = \dfrac{y}{2} = \dfrac{z}{5}$.

[**Ans.** : See Page No. 147, Q. No. 6.]

15. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\dfrac{\pi}{4}$ with x-axis, $\dfrac{\pi}{2}$ with y-axis and an acute angle θ with z-axis.

[**Ans.** : See Page No. 131, Q. No. 9.]

16. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared atleast once ?
[**Ans.** : See Page No. 167, Q. No. 4.]

17. A bag A contains 3 white and 2 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that the ball was drawn from the bag B.
[**Ans.** : See Page No. 169, Q. No. 9.]

18. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05, find the probability that out of 5 such bulbs
(i) none
(ii) not more than one
(iii) more than one
(iv) atleast one will fuse after 150 days of use.
[**Ans.** : See Page No. 175, Q. No. 4.]

Or

The cartesian equation of a line are $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$, write its vector form.

[**Ans.** : See Page No. 158, Q. No. 2.]

19. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains atleast 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below :

Food	*Vitamin A*	*Vitamin B*	*Vitamin C*
X	1	2	3
Y	2	2	1

1 kg of food X cost ₹ 16 and 1 kg of food Y cost ₹ 20. Find the least cost of the mixture which will produce require diet.

[**Ans.** : See Page No. 165, Q. No. 4.]

Or

A man is known to speak truth 3 times out of 4. He throws a die and reports that it is a six. Find the probability that it is actually a six.

[**Ans.** : See Page No. 168, Q. No. 4.]

Section-C

Question numbers **20** to **26** carry **6** marks each.

20. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f is invertible. Find also the inverse of f.

[**Ans.** : See Page No. 8, Q. No. 10.]

Or

Prove that : $2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

[**Ans.** : See Page No. 15, Q. No. 13.]

21. If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

[**Ans.** : See Page No. 27, Q. No. 4.]

22. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & , \text{ if } x \le 2 \\ ax + b & , \text{ if } 2 < x < 10 \\ 21 & , \text{ if } x \ge 10 \end{cases}$$

[**Ans.** : See Page No. 58, Q. No. 3.]

23. Find the equations of the tangent and normal to the curve $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

[**Ans.** : See Page No. 67, Q. No. 12.]

Or

Evaluate : $\int (x - 3)\sqrt{x^2 + 3x - 18}\, dx$.

[**Ans.** : See Page No. 84, Q. No. 1.]

24. Prove that : $\int_0^{\pi/2} \log\sin x\, dx = \int_0^{\pi/2} \log\cos x\, dx = -\frac{\pi}{2}\log 2$

[**Ans.** : See Page No. 95, Q. No. 2.]

25. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

[**Ans.** : See Page No. 105, Q. No. 4.] ·

26. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts while it takes 3 hour on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17.50 per package of nuts and ₹ 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit if he operates his machines for atmost 12 hours a day ?

[**Ans.** : See Page No. 161, Q. No. 5.]

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Series SSO

Code No. 65/1/RU

Examination Paper, 2015
C.B.S.E. (All India)
Mathematics
Class-XII
Set-I

Time : 3 *Hours*] [*M. M.* : 100

General Instructions **:**

(i) *All* questions are compulsory.
(ii) Please check that this question paper contains **26** questions.
(iii) Questions **1–6** in Section A are very short-answer type questions carrying **1** mark each.
(iv) Questions **7–19** in Section B are long-answer I type questions carrying **4** marks each.
(v) Questions **20–26** in Section C are long-answer II type questions carrying **6** marks each.
(vi) Please write down the serial number of the question before attempting it.

Section-A

Question numbers 1 to 6 carry 1 mark each.

1. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$.

2. Write the sum of the order and degree of the following differential equation :

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$$

3. Write the integrating factor of the following differential equation :

$$(1 + y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

4. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$.

5. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

6. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

Section-B

Question numbers 7 to 19 carry 4 marks each.

7. To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50
(ii) ₹ 20
(iii) ₹ 40

The number of attempts made in three villages X, Y, and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately, using matrices.
Write one value generated by the organisation in the society.

8. Solve for x : $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\frac{8}{31}$

Or

Prove the following : $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0$ $(0 < xy, yx, zx < 1)$

9. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4\,a^2b^2c^2.$$

10. Find the adjoint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence show that A. (adj A = $|A|$ I_3.

11. Show that the function $f(x) = |x-1| + |x+1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

12. If $y = e^{m \sin^{-1} x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$.

13. If $f(x)\ \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}.]$

14. Evaluate : $\int (3-2x).\sqrt{2+x-x^2}\,dx$

Or

Evaluate : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$

15. Find : $\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

16. Find : $\int \frac{\log x}{(x+1)^2}\,dx$

17. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

18. Find the equation of a line passing through the point (1, 2, – 4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

Or

Find the equation of the plane passing through the points (– 1, 2, 0), (2, 2, – 1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

19. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

Or

For 6 trials of an experiment, let X be a binomial variate which satisfies the relation 9P(X = 4) = P(x = 2). Find the probability of success.

Section-C

Question numbers 20 to 26 carry 6 marks each.

20. Consider $f : R_+ \to [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$.

Or

A binary operation $*$ is defined on the set $x = \mathrm{R} - \{-1\}$ by

$$x * y = x + y + xy, \ \forall \ x, y \in \mathrm{X}.$$

Check whether $*$ is commutative and associative. Find its identity element and also find the inverse of each element of X.

21. Find the value of p for when the curves $x^2 = 9p\,(9 - y)$ and $x^2 = p\,(y + 1)$ cut each other at right angles.

22. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4,$ $y = 4$, and $y = 0$ into three equal parts.

23. Show that the differential equation $\dfrac{dy}{dx} = \dfrac{y^2}{xy - x^2}$ is homogeneous and also solve it.

Or

Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that $x = 1$ when $y = 0$.

24. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, – 4, – 5) and B(2, – 3, 1) intersects the plane $2x + y + z = 7$.

25. A company manufactures three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is ₹ 12,000 and of factory II is ₹ 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost ? Formulate this problem as an LPP and solve it graphically.

26. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

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MODEL PAPER, 2016

Jharkhand Academic Council, Ranchi

Mathematics-XII

(Solved)

Time : 3 Hours] [*Full Marks* : 100; *Pass Marks* : 33

General Instructions :

(i) All questions are compulsory.

(ii) Candidates are required to give their answers in their own words as far as practicable.

(iii) The question paper consists of 29 questions divided into three Sections—*A*, *B* and *C*. Section-*A* comprises 10 questions of 1 mark each. Section-*B* comprises 12 questions of 4 marks each and Section-*C* comprises 7 questions of 6 marks each.

(iv) Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

Section-A : Objective Type Questions

1. What is the range of the function $f(x) = \dfrac{|x-1|}{x-1}$?

[**Ans.** : See Page No. 3, Q. No. 5.]

2. A binary composition $*$ is defined in Z^+ by $a * b = a^b - a$; $a, b \in Z^+$. Find $2 * 5$, where Z^+ is the set of positive integers.

[**Ans.** : See Page No. 5, Q. No. 15.]

3. Write the principal value of $\tan^{-1}\left[\sin\left(-\dfrac{\pi}{2}\right)\right]$.

[**Ans.** : See Page No. 11, Q. No. 7.]

4. Evaluate : $\Delta = \begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

[**Ans.** : See Page No. 35, Q. No. 1.]

5. If $y = \sin(\cot x)$, then find $\dfrac{dy}{dx}$.

[**Ans.** : See Page No. 49, Q. No. 8.]

6. Write the value of $\int \sec x \, dx$.

[**Ans.** : See Page No. 79, Q. No. 3.]

7. Evaluate : $\int \dfrac{1}{x^2}\, dx$.

[**Ans.** : See Page No. 80, Q. No. 9.]

8. Write the degree of the differential equation :

$$x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0$$

[**Ans.** : See Page No. 113, Q. No. 2.]

9. Find a vector in the direction of vector $\vec{a} = 3\vec{i} - 4\vec{j}$ that has magnitude 5 units.

[**Ans.** : See Page No. [illegible], Q. No. [illegible]]

10. If a line has direction ratios 2, – 1, – 2, determine its direction cosines.
[**Ans.** : See Page No. 145, Q. No. 8.]

Section-B

11. If in the set Q of all rational numbers, a binary operation O : Q × Q → Q, O $(x, y) = x + y - xy$ is defined, then prove that O is associative.
[**Ans.** : See Page No. 3, Q. No. 7.]

12. Is the function $f : R \to R$ onto function where $f(x) = 2x$? Give reasons.
[**Ans.** : See Page No. 4, Q. No. 12.]

13. Prove that : $\cos^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{33}{65} = \tan^{-1}\frac{56}{33}$
[**Ans.** : See Page No. 13, Q. No. 6.]

14. If $A + B = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $A - B = \begin{bmatrix} -3 & -6 \\ 4 & -1 \end{bmatrix}$, then find A.
[**Ans.** : See Page No. 25, Q. No. 5.]

15. If a, b, c are in A.P., then find the value of the determinant : $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$
[**Ans.** : See Page No. 37, Q. No. 1.]

Or

If $x = \cos\theta$ and $y = \sin^3\theta$, then prove that : $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\,(5\cos^2\theta - 1)$
[**Ans.** : See Page No. 52, Q. No. 11.]

16. If $(x - y)e^{\frac{x}{x-y}} = a$, Prove that : $y\frac{dy}{dt} + x = 2y$
[**Ans.** : See Page No. 51, Q. No. 7.]

Or

If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$; $y = a\sin t$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
[**Ans.** : See Page No. 53, Q. No. 14.]

17. If $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.
[**Ans.** : See Page No. 57, Q. No. 27.]

18. Using differentials, find the approximate value of $(3.968)^{3/2}$.
[**Ans.** : See Page No. 65, Q. No. 1.]

19. Find the equations of the tangent and normal to the curve $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.
[**Ans.** : See Page No. 67, Q. No. 12.]

20. Evaluate : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + a)}}$.
[**Ans.** : See Page No. 81, Q. No. 2.]

21. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$
[**Ans.** : See Page No. 93, Q. No. 8.]

Or

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.
[**Ans.** : See Page No. 103, Q. No. 4.]

22. Find the particular solution of the differential equation : $\frac{dy}{dx} = 1 + x + y + xy$, given that

[**Ans.** : See Page No. 115, Q. No. 5.]

Section-C

23. Let * be a binary operation on Q defined by : $a * b = \frac{3ab}{5}$.

Show that * is commutative as well as associa-tive. Also find its identify element, if it exists.
[**Ans.** : See Page No. 5, Q. No. 3.]

Or

Prove that : $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$
[**Ans.** : See Page No. 17, Q. No. 5.]

24. If $A = \begin{bmatrix} 0 & -\tan \alpha/2 \\ \tan \alpha/2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.
[**Ans.** : See Page No. 27, Q. No. 4.]

25. Using properties of determinants, prove the following : $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$

[**Ans.** : See Page No. 40, Q. No. 2.]

26. Show that the function $f(x) = |x - 3|, x \in$ IR is continuous but not differentiable at $x = 3$.
[**Ans.** : See Page No. 59, Q. No. 7.]

Or

A wire of length 28 cm is to be cut into two pieces. One of the piece is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum ?
[**Ans.** : See Page No. 71, Q. No. 2.]

27. Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$.
[**Ans.** : See Page No. 153, Q. No. 5.]

28. A gold smith manufactures necklace and bracelets the total no. of necklaces and bracelets that he can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. It is assumed that he can work for a maximum of 164 hrs. day. Further the profit on a bracelet is ₹ 300 and the profit on a necklace is 100. Find how many of each should be produced daily to maximize the total profit ?
[**Ans.** : See Page No. 159, Q. No. 1.]

29. There are two bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.
[**Ans.** : See Page No. 171, No. Q. 6.]

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EXAMINATION PAPER, 2015
Jharkhand Academic Council, Ranchi
Mathematics
Class-XII

Time : 3 *Hours*] [*Full Marks* : 100; *Pass Marks* : 33

General Instructions :

(i) All questions are compulsory.
(ii) Candidates are required to give their answers in their own words as far as practicable.
(iii) The question paper consists of 29 questions divided into three Sections—*A*, *B* and *C*. Section-*A* comprises 10 questions of 1 mark each. Section-*B* comprises 12 questions of 4 marks each and Section-*C* comprises 7 questions of 6 marks each.
(iv) Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

Section-A
Objective Type Questions

1. A binary composition $*$ is defined in Z^+ by $a * b = a^b - a; a, b \in Z^+$. Find $2 * 5$, where Z^+ is the set of positive integers.
2. Evaluate : $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
3. Construct a 3×2 matrix whose (i, j)th element $a_{ij} = \frac{1}{2}\ |i - 3j|$.
4. If $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, then find A^2.
5. Without expanding, prove that $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$.
6. Find the slope of the tangent to the curve $y = x^2 - x$ at $x = 2$.
7. Evaluate : $\int \frac{e^{\tan^{-1} x}}{1+x^2}\, dx$.
8. Find the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{i} \times \hat{j})$.
9. Find the area of a parallelogram whose adjacent sides are the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$.
10. Write the Cartesian equations of a straight line passing through the point (– 1, 2, 0) and whose direction ratios are 2, 4, – 3.

Section-B

11. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$.
12. Let $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$. Find the following :
(i) $f o f(7)$
(ii) $f o g(7)$
(iii) $g o f(7)$
(iv) $g o g(7)$

13. Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$.

14. Find the values of a and b such that the following function may be continuous everywhere :

$$f(x) = \begin{cases} 5 & ; \quad x \le 2 \\ ax+b & ; \quad 2 < x < 10 \\ 21 & ; \quad x \ge 10 \end{cases}$$

15. If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.

Or

If $y = \log\left(\sin\sqrt{x^2+1}\right)$, find $\frac{dy}{dx}$.

16. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

Or

Verify Rolle's theorem for the function $f(x) = x^2 - 4x + 3$ in the interval [1, 3].

17. Evaluate : $\int e^x \frac{(x-1)}{(x+1)^3} dx$.

18. Prove that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$.

19. Evaluate $\int_0^1 (x+x)^2 \, dx$ as a limit of a sum.

20. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, then find the following :

(i) $|\vec{a}|$

(ii) $\vec{a} \cdot \vec{b}$

(iii) $\vec{a} \times \vec{b}$

(iv) Projection of $\vec{b}$ on $\vec{a}$

21. Find the value of λ so that the four points with position vectors $-6\hat{i}+3\hat{j}+2\hat{k}$, $3\hat{i}+\lambda\hat{j}+4\hat{k}$, $5\hat{i}+7\hat{j}+3\hat{k}$ and $-13\hat{i}+17\hat{j}-2\hat{k}$ may be coplanar.

Or

Find the equation of the straight line which passes through the point (1, –3, 2) and is parallel to the straight line $\frac{-x-1}{3} = \frac{y+4}{1} = \frac{2z-4}{2}$.

22. Let A and B be two events such that $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$. Find $P(A \cup B)$.

Section-C

23. Using elementary row transformation, find the inverse of matrix A where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

Or

Solve the matrix method, the following equations :

$$x - 2y + z = 0$$
$$2x - y + z = 3$$
$$x + y + z = 6$$

24. At what points will the tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to the x-axis ? Also find the equations of the tangents to the curve at those points.

25. Using integration, find the area of the region bounded by $x = 2$ and $y^2 = 8x$.

26. Solve : $(x^3 + y^3)\, dy - x^2\, y dx = 0$.

Or

Solve : $(1 + x^2)\dfrac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$.

27. Consider the equations of the straight lines given by

$L_1 : \vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$.

$L_2 : \vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$.

If $\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b_1} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$, then find

(i) $\vec{a_1} \times \vec{a_2}$

(ii) $\left(\vec{a_1} \times \vec{a_2}\right) \cdot \left(\vec{a_2} - \vec{a_1}\right)$

(iii) the shortest distance between L_1 and L_2.

28. Urn A contains 1 white, 2 black and 3 red balls. Urn B contains 2 white, 1 black and 1 red balls and Urn C contains 4 white, 5 black and 3 red balls. One urn is selected at random and two balls are drawn. Let E be the event that the balls drawn are one white and one red and E_1, E_2, E_3 be events that the selected urns are A, B and C respectively. Now find the following.

(i) $P(E_1)$

(ii) $P(E_2)$

(iii) $P(E/E_3)$

(iv) $P(E/E_1)$

29. Solve graphically the following L.P.P. :

Minimize	$Z = -3x + 4y$
subject to	$x + 2y \le 8$
	$3x + 2y \le 12$
and	$x, y \ge 0$.

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